

Mathematical Proficiency

By Kenneth Danley

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We know that parents are eager to help their children be successful in school. Many parents spend countless hours reading to and with their children. This is one of the greatest contributions a parent can make to their child's education. The research is clear; these actions improve student reading skills. Reading is fundamental, but so is math. When it comes to math, parents may not always be sure how to go about helping their children. I would bet every parent has heard this a few times, "That's not the way my teacher says to do it." That may or may not be true, but parents can help when they understand some current basic ideas regarding what math instruction should include, as well as, how to approach the concept. I would like to help our community understand some of the findings of the National Research Council's Mathematics Learning Study Committee.



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Let's start with the idea that mathematical proficiency requires five stands:

- (1) Understanding
- (2) Computing
- (3) Applying
- (4) Reasoning
- (5) Engaging

Knowing what each strand means is very important and we will address them all over the course of this column, but for now it is critical that everyone understand that these five strands are interwoven and interdependent. In the past some math advocates have stressed only one aspect of math proficiency. This approach may provide some success, however there can be

limitations. The National Research Council believes that all five strands should be a part of math instruction at all levels.

All parents must remember that open and honest communication with their child's teacher is crucial for student success. We can know a great deal about learning, but without choreography among students, parents, and teachers the plan may not achieve its full potential. Keep those lines of cooperative communication open!

Understanding

The first strand is **Understanding**. Understanding refers to a student's grasp of the basic fundamentals of math. Students must clearly understand what mathematical symbols and procedures mean. It is important for our students to know basic computation facts, but it is also critical that students are able to explain why the answer is so and support their conclusion with a clear explanation. Explaining an answer makes the student consciously think of the solution which builds the understanding of the math and how it applies to the situation, not just recall an isolated fact. In addition to the ability to explain an answer, students must also be able to see mathematical relationships between procedures. The connection between addition and multiplication are easily illustrated by adding same-size groups of counters. Subtraction and division are also related when we consider the process of removing same-size groups from a larger total and recording the number of times a group is removed. Another important concept is opposite functions show relationships. Subtraction is the opposite function of addition and when an action uses circumstances and wording to support

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subtraction, the same set of circumstances do not usually support addition. We must also consider that students who know their multiplication facts have much less difficulty with division computation. It takes time for these relationships to develop in young minds and it requires steady practice. Always remember to ask, "How did you get that answer?" and wait for a clear explanation. Then ask if there is another way to get the same answer.

Understanding helps students construct a foundation for remembering facts and skills for solving new and unfamiliar problems. In addition, a good understanding helps students avoid simple errors, and allows them to determine if the answer makes sense.

We mentioned that understanding requires more than computation, it also includes the idea that students should be able to "see" the mathematics. Parents can help by showing that the numerical representation of the addition problem $5+3 = 8$ can be demonstrated with 5 chips combined with 3 chips makes a total of 8 chips.

$$\bullet\bullet\bullet\bullet\bullet + \bullet\bullet\bullet = \bullet\bullet\bullet\bullet\bullet\bullet\bullet\bullet$$

Then ask the student to make other addition sequences using a total of 8 chips. Soon a child will see that addition isn't just rote memorization of a fact, but an actual action which can be observed and manipulated. If 5 chips are added to 3 chips, and that equals a total of 8 chips, then the child may also see that 2 chips added to 2 chips added to 4

chips also equals 8 chips. This is powerful learning for a young mind just developing the concept of addition. The same actions can be used to demonstrate the concept of subtraction. Multiplication is another easily demonstrated concept. Most children learn their multiplication tables by memorization, much the same way they learn a phone number. Having the multiplication tables memorized is a good thing, but we can make that learning more powerful by actually showing the multiplication sentence 3×5 is three groups of five counters. We might

easily demonstrate that 4×5 is three groups of five counters plus another group of five

counters. This can be accomplished using grid or graph paper and make groups of five squares. Additional groups may be constructed and then the multiplication sentence created. Since multiplication is the opposite of division we can

take a group of counters or squares on grid paper and begin to demonstrate what division actually means.

20 counters can be divided into 4 groups of 5. In every case, each action of moving counters should be accompanied with the student writing the number sentence and mathematical symbols, as well as explaining what they are doing.

Whenever we wish to help children learn the concept of a math function it is important that they are able to manipulate objects and actually see the mathematics take place.

Computing

Most of us remember our math instruction

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as lots of practice with procedures and crunching numbers. Recently we have heard that math instruction should include much more than computation, and we must agree, but let's not discount how important computational skills are to a complete math education. Procedures for addition, subtraction, multiplication and division of whole numbers, decimals, and fractions need to be completed efficiently and accurately. Practice develops this skill, but a complete understanding as discussed in the previous section helps the student recognize and avoid common errors. Just as students who struggle with reading fluency use enormous amounts of energy and concentration just to make sense of a passage lose motivation to read, the same holds true for math. A student who is not fluent in computation can easily become discouraged in their effort to perform mathematical tasks and lose the motivation to see that math is predictable and well structured. So practice in computational procedures is necessary. Add in the understanding component and we are well on our way to overall mathematical proficiency.

There are several things parents can do to help develop computational skills. Obviously practice with math procedures, (algorithms), is time well spent. Add meaning to the computation by asking your child to explain each step. Don't over practice a skill. Yes, practice makes perfect, but over practice takes time away from other important practice. Computing does imply procedural skills, but it also includes measurement, constructing figures common in geometry, and even simple graphing of data.

One last comment before we leave the area

of computing. When students merely memorize procedures they lose the ability to apply these skills in new and unique problem solving situations. Memorizing procedures is good, but not as the end of the instruction. We must continue to build student understanding as well, and the two must be taught together.

Applying

As we weave this strand into the fabric of mathematics instruction we want to remember this is where the rubber meets the road. Applying involves using computational skills and mathematics understanding to solve routine and non-routine problems. We often see a page of math problems set up in neat rows and students solve each. This is good skill practice, but students will likely encounter situations that require them to determine what information is needed to solve the problem and what information should be ignored. To achieve that end students must apply their understanding of the problem and compute correctly for a solution. Students at a higher level need to assess the situation, pose a problem, and develop strategies for a solution.

Parents can do a great deal to help students apply their knowledge by taking real life situations and asking for a solution. One example may have a student read in the newspaper that bananas are three pounds for a dollar, while at a different store the bananas are thirty five cents a pound. The task for the student is to find which is the best price per pound and explain the reasoning. There are a number of ways we can determine the price per pound and we would lead our children toward a solution. This is a simple example and we can

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increase the difficulty appropriate to the student's age. The key is to use the math often in these real life situations. Also, it can be very helpful for students to create visual representations for the math as they construct solutions. Another helpful parent strategy is ask your child what they learned in school that day, and then ask them to explain how it might be applied in real life. Parents should also expect their child's homework to include more than simple computation. There may be some confusion from time to time, but always help your child realize that math is understandable to everyone.

Just as every parent strives to read with their children, we must strive to work real life math into their everyday lives as well.

Reasoning

One of the most useful consequences of studying mathematics is the ability to see that there are numerous relationships between mathematical concepts and make use of those relationships to apply problem solving strategies. This is **Reasoning**, our fourth strand of mathematical proficiency. In advanced mathematics, reasoning is used to prove that specific statements are true and a required explanation or proof is expected. At a more basic level, mathematics reasoning requires students to recognize that mathematics makes sense and can be understood. They learn how to evaluate situations, select a problem solving strategy and explain solutions. We often see arithmetic problems carefully arranged so that computation is all that is needed for a solution, (we refer to

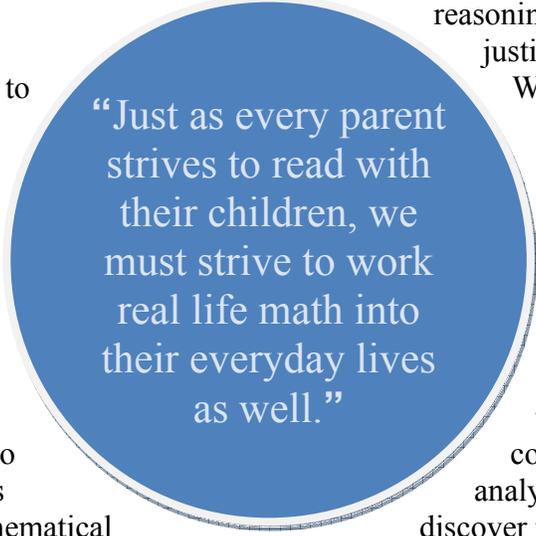
these arrangements as algorithms), but students must see that there is meaning in the algorithm. An algorithm is an efficient rule for solving an equation based on mathematical reasoning. Students who have mastered the mathematical reasoning concept are able to reflect on solutions to problems and determine whether or not they make sense. They can see the relationship of the computation to what is called for in the problem solving situation.

One of the best ways to help your child become proficient in mathematical reasoning is to have them explain or justify their solutions to you.

When we look at the algorithm, $5 \times 6 = 30$ why do we get the same answer when we use the algorithm $5 \times 3 \times 2 = 30$? Most adults understand the idea that $2 \times 3 = 6$ and $5 \times 6 = 30$, but for a child it might require additional thought about the computation. As students analyze the problem they may discover the relationship between

2×3 and 6. Take the idea beyond just understanding the connection; require they explain their discovery. Parents can facilitate this development of mathematical reasoning by carefully listening to student explanations and accept only those that make good sense. In the process of explaining the reasoning, students sharpen their reasoning skills as well as learn to see a solution to a problem as sensible and related to the demands of the situation.

The relationship of the strands **Reasoning** and **Understanding** is very strong and may seem to overlap. We would agree that the



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similarities are compelling, but reasoning requires more justification as to why the answer is so. This will prepare our young students for the actual proofs required at higher levels of mathematics.

Engaging

When we consider the last of the five mathematical proficiency strands, **Engaging**, we can say that students who are “engaged” in the mathematical learning see math as sensible, worthwhile, and are willing to put effort into their mathematics learning. Perhaps the strongest element of the engaging strand is student attitude. A student’s attitude toward math has a great effect on their achievement. A positive attitude toward math engages students to see that math can be usefully applied in a number of real life situations. Negative attitudes toward math, or anything for that matter, are learned and impact student initiative and willingness. How many times have you heard someone dismiss a mathematical task as too difficult and say, “I never was any good at math anyway.” Can you imagine someone saying that about reading? The point here is that children hear our comments, watch our actions and develop attitudes based on these comments. As parents and teachers we must demonstrate to our children the effectiveness of mathematics to elevate its importance. As students learn and understand mathematics the entire concept becomes more sensible to them.

Once again parents can contribute to this strand by demonstrating that we are surrounded by mathematics. At the gas station we might fill the tank up and realize we needed ten gallons to go two hundred miles. Parents can ask about fuel efficiency

by showing that total miles traveled divided by total gallons can tell us our miles per gallon. We could even go farther by asking what it cost to drive one mile. This is a good place to use calculators and demonstrate the common sense of mathematics while avoiding the drudgery of long calculations. Just remember, no matter what the mathematical task, keep a positive attitude and show it can be done.

It is very important that we restate the idea that all of these strands work together to make superior mathematics instruction for our children. Just as a table needs all of its legs to function correctly, mathematics instruction needs all of the strands to effectively reach the goal of mathematically proficient students.